Inventory Model for Deteriorating Items Having Two Component Mixture of Pareto Lifetime and Selling Price Dependent Demand

Vijayalakshmi. G, Srinivasa Rao. K and Nirupama Devi. K

Abstract — In this paper, we develop and analyses an inventory model with the assumption that the life time of commodity is random and follows two component mixture of Pareto distribution. It is also assumed that the demand is a function of selling price. Using the differential difference equations, the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function per a unit time and the profit rate function are obtained. Sensitivity analysis is also carried out for this model. This model is useful in practical situations like food processing industries, market yards dealing with agricultural products.

Key words — demand, deteriorating items, life time, mixture distributions, Pareto distribution, selling price.

1 INTRODUCTION

INVENTORY modeling is a prerequisite for developing the optimal ordering and pricing policies in several places such as ware houses, market yards, production processes, cargo handling. When one is concerned with the inventory control, it is essential to identify two important factors namely, the nature of the goods procured and the nature of the demand. When we consider the nature of procurement, we have to consider life time of the commodity along with the demand pattern (K.Srinivasa Rao et.al [9], K.V Subbiah et.al [14]). Decay or deterioration of physical goods while in stock is a common phenomenon. . Pharmaceuticals, food, vegetables and fruits are a few examples of such items. Taking this into consideration, deteriorating inventory models have been widely studied in recent years. The analysis of decaying inventory problems began with Ghare, P.M and Schrader,G.F [2] who developed an economic order quantity model with constant rate of decay (exponential life time). Covert R.P and Philip, G.C [1] extended this model and obtained an economic order quantity model for a variable rate of deterioration by assuming a two parameter Weibull distribution. Extensive reviews of research literature on the deteriorating items are provided by Raafat, F [7], Goyal S.K. and Giri B.C [3] and Liao. et al [6].

Very little work has been done in perishable inventory models with heterogeneous life time of the commodity. Several authors have studied the perishable inventory models with the assumption that the life time of the commodity is random and follows a probability distribution having the homogenous nature.

However in some situations prevailing at places like fruit and vegetable markets, food processing industries, photo chemicals, pharmaceutical industries, the stock on-hand is procured from various sources for the same type of items. Even though the nature of the item does not differ, there will be some inherent variation due to the source from which they are procured. So the efficiency of the inventory model heavily depends upon the probability distribution ascribed to the life time of the commodity under consideration. In general the stock on-hand with respect to the perishability can be considered as a heterogeneous population consisting of two types of life times viz., shorter and longer life times. When the items are mixed in stock, it is difficult to isolate each item with respect to their life time. So in order to associate suitable probability model to the life time of the commodity one has to consider the mixture of distributions (Srinivasa Rao et al [10].

Hence in this paper, we consider the life time of the commodity is random and follows a two component Pareto mixture of distributions. The Pareto life time distribution is capable of characterizing age dependent rate of deterioration. The Pareto distribution is extensively used in reliability and life testing experiments.

In addition to the life time of the commodity, another important factor that influences the inventory system is the nature of the demand. Several demand patterns were used for analyzing the inventory situations (Srinivasa Rao et al [10]). Among all these demand patterns selling price dependent demand gained lot of importance in developing and analyzing the inventory models (Lakshmi Narayana et al. [5], Srinivasa Rao et al. [11], Sridevi et al [8], U.M.Rao et al [13], Essey et al. [4]). In this article also it is assumed that the demand is a linear function of selling price.

Using the differential difference equations the instantaneous state of inventory is derived with suitable cost considerations. The total cost function per a unit time and the profit rate function are derived. Through numerical illustration the sensitivity of the model with respect to the cost is also studied. This model is much useful for analyzing inventory situations arising at the food processing industries, market yards dealing with agricultural products.

- Vijayalakshmi. G, Researche Scholar
  E-mail: gullipallivijaya@gmail.com
- Srinivasa Rao. K, Professor, E-mail: ksraoau@yahoo.co.in
- Nirupama Devi. K, Professor,
  E-mail: knirupamadevi@gmail.com
  Dept.of Statistics, Andhra University, Visakhapatnam, India.

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2 INVENTORY MODEL FOR DETERIORATING ITEMS WITH SELLING PRICE DEPENDENT DEMAND

Consider an inventory system for deteriorating items in which the life time of the commodity is random and follows two component mixture of Pareto distribution with probability density function of the form

$$f(t) = \alpha\beta_1\theta^{\beta_1}t^{-(\beta_1+1)} + (1-\alpha)\beta_2\theta^{\beta_2}t^{-(\beta_2+1)} \qquad t \geq \theta$$

The distribution function of t is

$$F(t) = \alpha\left[1-\left(\frac{\theta}{t}\right)^{\beta_1}\right] + (1-\alpha)\left[1-\left(\frac{\theta}{t}\right)^{\beta_2}\right]$$

The mean life time of the commodity is

$$\mu = \alpha\frac{\beta_1\theta}{(\beta_1-1)} + (1-\alpha)\frac{\beta_2\theta}{(\beta_2-1)} \qquad t \geq \theta,\ \beta_1 > 1,\ \beta_2 > 1$$

The variance of life time of the commodity is

$$Var(t) = \alpha\frac{\beta_1\theta^2}{\beta_1-2} + (1-\alpha)\frac{\beta_2\theta^2}{\beta_2-2} - \left[\alpha\frac{\beta_1\theta}{(\beta_1-1)} + (1-\alpha)\frac{\beta_2\theta}{(\beta_2-1)}\right]^2 \quad t \geq \theta,\ \beta_1 > 2, \beta_2 > 2$$

The instantaneous rate of deterioration is

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\alpha\beta_1\theta^{\beta_1}t^{-(\beta_1+1)} + (1-\alpha)\beta_2\theta^{\beta_2}t^{-(\beta_2+1)}}{1-\ \alpha\left[1-\left(\frac{\theta}{t}\right)^{\beta_1}\right] - (1-\alpha)\left[1-\left(\frac{\theta}{t}\right)^{\beta_2}\right]} \qquad (2.1)$$

3.1 INVENTORY MODEL FOR DETERIORATING ITEMS WITH SELLING PRICE DEPENDENT DEMAND WITH SHORTAGES

Consider an inventory system for deteriorating items in which the life time of commodity is random and follows a two component mixture of Pareto distribution with probability density of the form as given in section 2.

Assumptions:

The demand rate is a function of unit selling price
Lead time is zero
Cycle length, T is unknown
Shortages are allowed
A deteriorated unit is lost

Notation:

Q: Ordering quantity in one cycle
A: Ordering cost
C: Cost per unit
h: Inventory holding cost per unit per unit time
π: Shortage cost per unit per unit time
s: Selling price of a unit
$\lambda(s)$: Demand rate

In this model the stock level (initial inventory) is Q at time $t = 0$. The stock level decreases during the period $(0,\theta)$ due to demand and due to demand and deterioration during the period (θ, t_1) . At time t_1 inventory reaches zero and back orders accumulate during the period (t_1, T). Let $I(t)$ be the inventory level of the system at time t $(0 \leq t \leq T)$

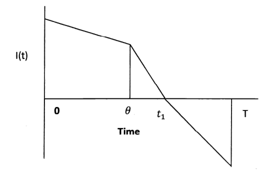

Fig.1 : Schematic diagram representing the inventory level

The differential equations governing the instantaneous state of $I(t)$ over the cycle length T are

$$\frac{d}{dt}I(t) = -\lambda(s) \qquad 0 \leq t \leq \theta \qquad (3.1.1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\lambda(s) \qquad \theta \leq t \leq t_1 \qquad (3.1.2)$$

$$\frac{d}{dt}I(t) = -\lambda(s) \qquad t_1 \leq t \leq T \qquad (3.1.3)$$

where, $h(t)$ is as given in equation (2.1),with the initial conditions $I(0) = Q$, $I(t_1) = 0$.
Substituting $h(t)$ given in equation (2.1) in equations (3.1.1), (3.1.2), and (3.1.3) and solving the above differential equations, the on hand inventory at time t can be obtained as

$$I(t) = \lambda(s)\left\{(\theta - t) + \int_\theta^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right) - (1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right\} \quad 0 \leq t \leq \theta \qquad (3.1.4)$$

$$I(t) = \lambda(s)\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right) - (1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right] \int_t^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right) - (1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt \quad \theta \leq t \leq t_1 \qquad (3.1.5)$$

$$I(t) = \lambda(s)(t_1 - t) \qquad t_1 \leq t \leq T \qquad (3.1.6)$$

The stock loss due to deterioration in the interval $(0,t)$ is

$$L(t) = \lambda(s)\left\{(\theta - t) + \int_\theta^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right) - (1-\alpha)\right.\right.$$

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$$\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\Bigg]^{-1} dt - \left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$\int_t^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} du\Bigg\}$$

The stock loss due to deterioration in the interval $(0, t_1)$ is

$$L(t_1) = \lambda(s)\Bigg\{(\theta - t_1) + \int_\theta^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)$$

$$\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\Bigg]^{-1} dt\Bigg\} \qquad (3.1.7)$$

The ordering quantity, Q in the cycle length T is

$$Q = L(t_1) + \lambda(s)t_1 + \lambda(s)(T - t_1)$$

$$= \lambda(s)\Bigg\{\theta + (T - t_1) + \int_\theta^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)$$

$$-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\Bigg]^{-1} dt\Bigg\} \qquad (3.1.8)$$

Let $K(t_1, T, s)$ be the total cost per unit time. Since the total cost is sum of the set up cost, cost of the units, the inventory holding cost, the shortage cost, $K(t_1, T, s)$ becomes

$$K(t_1,T,s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^\theta I(t)dt + \int_\theta^{t_1} I(t)dt\right] + \frac{\pi}{T}\int_{t_1}^{T} -I(t)dt \qquad (3.1.9)$$

Substituting the values of $I(t)$ and Q given in equations (3.1.4), (3.1.5), (3.1.6) and (3.1.8) in equation (3.1.9), we obtain $K(t_1, T, s)$ as

$$K(t_1,T,s) = \frac{A}{T} + \frac{\lambda(s)C}{T}\Bigg\{\theta + (T - t_1)\int_\theta^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)$$

$$-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} dt\Bigg\} + \frac{\lambda(s)h}{T}\int_0^\theta\Bigg\{(\theta - t) + \int_\theta^{t_1}\Bigg[1-\alpha$$

$$\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\Bigg\}dt + \frac{\lambda(s)h}{T}$$

$$\int_\theta^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\Bigg\}dt$$

$$\int_t^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\quad\Bigg\}dt$$

$$+\frac{\lambda(s)\pi}{T}(T - t_1) \qquad (3.1.10)$$

Let $P(t_1, T, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit time minus total cost per unit time, we have

$$P(t_1,T,s) = s\lambda(s) - K(t_1,T,s) \qquad (3.1.11)$$

Substituting the value of $K(t_1, T, s)$ given in equation (3.1.10) in equation (3.1.11), we obtain the profit rate function as

$$P(t_1,T,s) = s\lambda(s) - \frac{A}{T} - \frac{C\lambda(s)}{T}\Bigg\{\theta + (T - t_1) + \int_\theta^{t_1}\Bigg[1-\alpha$$

$$\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\Bigg\}$$

$$-\frac{h\lambda(s)}{T}\int_0^\theta\Bigg\{(\theta - t) + \int_\theta^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)$$

$$\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\Bigg\}dt - \frac{h\lambda(s)}{T}\int_\theta^{t_1}\Bigg\{\Bigg[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)$$

$$-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\Bigg]\int_t^{t_1}\Bigg[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)$$

$$-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\Bigg]^{-1} du\quad\Bigg\}dt + \frac{\pi}{2T}(T - t_1)^2 \qquad (3.1.12)$$

3.2 OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section we obtain the optimal policies of the inventtory system developed in section 3.1. The conditions for obtaining optimality are

$$\frac{\partial}{\partial t_1}P(t_1,T,s) = 0; \quad \frac{\partial}{\partial T}P(t_1,T,s) = 0; \quad \frac{\partial}{\partial s}P(t_1,T,s) = 0;$$

$$D = \begin{vmatrix} \frac{\partial^2}{\partial t_1^2}P(t_1,T,s) & \frac{\partial}{\partial t_1\partial T}P(t_1,T,s) & \frac{\partial}{\partial t_1\partial s}P(t_1,T,s) \\ \frac{\partial}{\partial T\partial t_1}P(t_1,T,s) & \frac{\partial^2}{\partial T^2}P(t_1,T,s) & \frac{\partial}{\partial T\partial s}P(t_1,T,s) \\ \frac{\partial}{\partial s\partial t_1}P(t_1,T,s) & \frac{\partial}{\partial s\partial T}P(t_1,T,s) & \frac{\partial^2}{\partial s^2}P(t_1,T,s) \end{vmatrix} < 0$$

where D is the determent of Hessian matrix

Differentiating (3.1.12) with respect to t_1 and equating to zero, we get

$$c\Bigg\{1-\Bigg[1-\alpha\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_2}\right)\Bigg]^{-1}\Bigg\}$$

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$$-h\int_0^{\theta}\left[1-\alpha\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t_1}\right)^{\beta_2}\right)\right]^{-1}dt$$

$$-h\int_{\theta}^{t_1}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}\right\}dt$$

$$-\pi(T-t_1)=0 \qquad (3.2.1)$$

Differentiating (3.1.12) with respect to T and equating to zero, we get

$$\frac{A}{\lambda(s)}-C\left[(t_1-\theta)+\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\right.\right.$$

$$\left.\left.\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right]+h\int_0^{\theta}\left\{(\theta-t)+\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)\right.\right.$$

$$\left.\left.-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$+h\int_{\theta}^{t_1}\left\{\left(\int_{t}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right.\right.$$

$$\left.\left.\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right)\right\}dt$$

$$+\pi(T^2-t_1^{\,2})=0 \qquad (3.2.2)$$

Differentiating (3.1.12) with respect to s and equating to zero, we get

$$s+\frac{\lambda(s)}{\lambda'(s)}-\frac{c}{T}\left\{\theta+(T-t_1)\right.$$

$$\left.+\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}$$

$$-\frac{h}{T}\int_0^{\theta}\left\{(\theta-t)+\int_{\theta}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)\right.\right.$$

$$\left.\left.-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$-\frac{h}{T}\int_{\theta}^{t_1}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\int_{t}^{t_1}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$-\frac{\pi}{2T}(T-t_1)^2=0 \qquad (3.2.3)$$

Solving equations (3.2.1), (3.2.2) and (3.2.3) simultaneously, we obtain the optimal cycle length, T^*, time at which shortages occur t_1^* and optimal selling price, s^*.

The optimal ordering quantity, Q^* is obtained by substituting the optimal values of t_1, T and s in equation (3.1.8).

$$Q^*=\lambda(s^*)\left\{\theta+(T^*-t_1^{\,*})\right.$$

$$\left.+\int_{\theta}^{t_1^{\,*}}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}dt\right\} \qquad (3.2.4)$$

3.3 SENSITIVITY ANALYSIS

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal values of time at which shortages occur, the cycle length, selling price, ordering quantity. by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 1. The relationship between the parameters and the optimal values are shown in Fig 2.

Table 1
Sensitivity analysis of the model – with shortages

Variation Parameter	Optimal Values	% Change in Parameter						
		-15	-10	-5	0	5	10	15
A	t_1^*	4.1427	4.1502	4.1576	4.1649	4.1721	4.1791	4.1860
	T^*	5.6863	5.7656	5.8438	5.9209	5.9971	6.0722	6.1464
	s^*	102.5790	102.5834	102.5878	102.5922	102.5967	102.6013	102.6059
	K^*	281.1471	282.8823	284.5937	286.2821	287.9486	289.5938	291.2187
	P^*	4715.5272	4713.7808	4712.0580	4710.3580	4708.6799	4707.0228	4705.3860
	Q^*	277.4161	281.3137	285.1588	288.9537	292.7002	296.4002	300.0554
C	t_1^*	4.1905	4.1811	4.1726	4.1649	4.1579	4.1515	4.1455
	T^*	5.9377	5.9313	5.9258	5.9209	5.9167	5.9129	5.9096
	s^*	102.2173	102.3423	102.4673	102.5922	102.7172	102.8422	102.9672
	K^*	250.5002	262.4616	274.3888	286.2821	298.1419	309.9685	321.7621
	P^*	4747.0415	4734.7952	4722.5675	4710.3580	4698.1664	4685.9924	4673.8357
	Q^*	291.0828	290.3254	289.6182	288.9537	288.3259	287.7298	287.1615
h	t_1^*	4.1680	4.1670	4.1659	4.1649	4.1639	4.1629	4.1618
	T^*	5.9107	5.9141	5.9175	5.9209	5.9243	5.9278	5.9312
	s^*	102.5883	102.5896	102.5909	102.5922	102.5935	102.5948	102.5961
	K^*	285.9689	286.0734	286.1778	286.2821	286.3863	286.4903	286.5943
	P^*	4710.6814	4710.5735	4710.4657	4710.3580	4710.2505	4710.1431	4710.0358
	Q^*	288.4880	288.6433	288.7985	288.9537	289.1087	289.2637	289.4185
π	t_1^*	4.1603	4.1620	4.1635	4.1649	4.1662	4.1674	4.1686
	T^*	6.1747	6.0820	5.9977	5.9209	5.8506	5.7859	5.7261
	s^*	102.5955	102.5944	102.5933	102.5922	102.5912	102.5902	102.5892
	K^*	285.2054	285.5925	285.9503	286.2821	286.5907	286.8786	287.1477
	P^*	4711.4262	4711.0421	4710.6871	4710.3580	4710.0521	4709.7668	47095002
	Q^*	301.2713	296.7688	292.6816	288.9537	285.5389	282.3988	279.5010
θ	t_1^*	3.5592	3.7615	3.9634	4.1649	4.3661	4.5670	4.7677
	T^*	5.4886	5.6290	5.7732	5.9209	6.0719	6.2259	6.3826
	s^*	102.6082	102.6021	102.5968	102.5922	102.5883	102.5850	102.5821
	K^*	290.4564	288.9680	287.5791	286.2821	285.0703	283.9372	282.8769
	P^*	4706.1423	4707.6466	4709.0493	4710.3580	4711.5800	4712.7218	4713.7895
	Q^*	267.9135	274.7482	281.7661	288.9537	296.2982	303.7881	311.4126
β_1	t_1^*	4.1772	4.1729	4.1688	4.1649	4.1612	4.1577	4.1543
	T^*	5.9316	5.9279	5.9243	5.9209	5.9177	5.9147	5.9117
	s^*	102.5925	102.5924	102.5923	102.5922	102.5922	102.5921	102.5920
	K^*	286.2421	286.2561	286.2695	286.2821	286.2942	286.3057	286.3167
	P^*	4710.3975	4710.3837	4710.3705	4710.3580	4710.3461	4710.3348	4710.3239
	Q^*	289.5167	289.3190	289.1316	288.9537	288.7844	288.6233	288.4696
β_2	t_1^*	4.1795	4.1743	4.1695	4.1649	4.1606	4.1565	4.1526
	T^*	5.9337	5.9292	5.9249	5.9209	5.9172	5.9136	5.9102
	s^*	102.5925	102.5924	102.5923	102.5922	102.5922	102.5921	102.5920
	K^*	286.2353	286.2518	286.2674	286.2821	286.2961	286.3093	286.3218
	P^*	4710.4041	4710.3879	4710.3725	4710.3580	4710.3443	4710.3313	4710.3189
	Q^*	289.6233	289.3863	289.1635	288.9537	288.7555	288.5682	288.3907
α	t_1^*	4.1679	4.1669	4.1659	4.1649	4.1639	4.1630	4.1620
	T^*	5.9235	5.9226	5.9218	5.9209	5.9201	5.9193	5.9184
	s^*	102.5923	102.5923	102.5923	102.5922	102.5922	102.5922	102.5922
	K^*	286.2726	286.2758	286.2790	286.2821	286.2852	286.2883	286.2914
	P^*	4710.3675	4710.3643	4710.3611	4710.3580	4710.3550	4710.3519	4710.3489
	Q^*	289.0888	289.0433	288.9982	288.9537	288.9096	288.8660	288.8228

International Journal of Scientific & Engineering Research Volume 5, Issue 7, July-2014
ISSN 2229-5518

Variation Parameter	Optimal Values	% Change in Parameter						
		-15	-10	-5	0	5	10	15
a	t_1^*	4.1904	4.1811	4.1726	4.1649	4.1578	4.1513	4.1452
	T^*	6.1935	6.0937	6.0033	5.9209	5.8456	5.7765	5.7127
	s^*	87.6089	92.6026	95.5971	102.5922	107.5879	112.5840	117.5805
	K^*	247.2383	260.2829	273.2963	286.2821	299.2435	312.1830	325.1030
	P^*	3361.8586	3786.3302	4235.8312	4710.3580	5209.9080	5734.4785	6284.0676
	Q^*	255.8047	266.9181	277.9652	288.9537	299.8901	310.7801	321.6283
b	t_1^*	4.1643	4.1645	4.1647	4.1649	4.1651	4.1653	4.1655
	T^*	5.9148	5.9169	5.9189	5.9209	5.9230	5.9250	5.9271
	s^*	120.2389	113.7031	107.8553	102.5922	97.8304	93.5016	89.5491
	K^*	287.2908	286.9546	286.6184	286.2821	285.9458	285.6094	285.2730
	P^*	5592.2071	5265.5776	4973.3479	4710.3580	4472.4308	4256.1486	4058.6882
	Q^*	289.8058	289.5218	289.2378	288.9537	288.6695	288.3853	288.1010

As the ordering cost A increases the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal selling price s^*, and optimal quantity Q^*, are increasing and profit per unit time P^*, is decreasing. As the cost per unit C increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, and profit per unit time P^*, and optimal ordering quantity Q^* are decreasing and optimal selling price s^*, is increasing.

As the holding cost h increases the optimal time at which shortages occur t_1^*, and profit per unit time P^*, are decreasing and the optimal cycle length T^*, optimal selling price s^*, optimal ordering quantity Q^* are increasing.

As the shortage cost π increases, the optimal time at which shortages occur t_1^* increases and optimal cycle length T^*, optimal selling price s^*, optimal profit per unit time P^*, optimal ordering quantity Q^* are decreasing. As the scale parameter θ increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, and profit per unit time P^*, and optimal ordering quantity Q^* are decreasing and optimal selling price s^*, is decreasing.

As the shape parameter β_1 increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal selling price s^*, optimal profit per unit time P^*, optimal ordering quantity Q^* are decreases. As the shape parameter β_2 increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal selling price s^*, optimal profit per unit time P^*, optimal ordering quantity Q^* are decreases.

As the mixing parameter α increases, optimal time at which shortages occur t_1^*, the optimal cycle length T^*, optimal selling price s^*, optimal profit per unit time P^*, optimal ordering quantity Q^* are decreasing. As the demand parameter a increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, are decreases and optimal selling price s^*, optimal profit per unit time P^*, and optimal ordering quantity Q^* are increasing. As the demand parameter ' b ' increases, the optimal time at which shortages occur t_1^*, the optimal cycle length T^*, are increasing and optimal selling price s^*, optimal profit per unit time P^*, and optimal ordering quantity Q^* are decreasing.

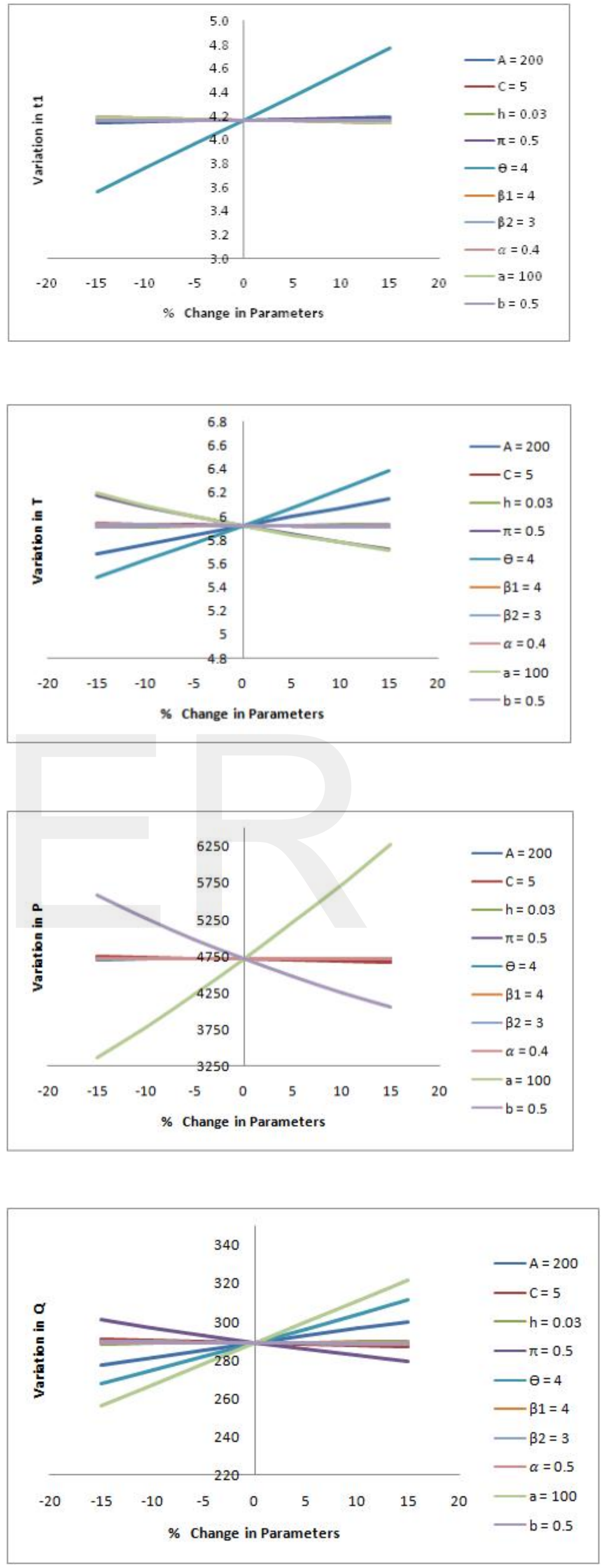


Fig 2: Relationship between optimal values and parameters with shortages

International Journal of Scientific & Engineering Research Volume 5, Issue 7, July-2014
ISSN 2229-5518

4.1 INVENTORY MODEL FOR DETERIORATING ITEMS WITH SELLING PRICE DEPENDENT DEMAND WITHOUT SHORTAGES

In section 3, the inventory model for deteriorating items with selling price dependent demand and as a function of selling price and with shortages is discussed. In this section the model without shortages is developed and analyzed. For developing the model, we assume that $\pi \to \infty$ and $t_1 \to T$.

In this model the stock level (initial inventory) is Q at time $t = 0$. The stock level decreases during the period $(0, \theta)$ due to demand and due to deterioration and demand during the period (θ, T). At time T inventory reaches zero.

The differential equations governing the instantaneous state of $I(t)$ over the cycle length T are

$$\frac{d}{dt} I(t) = -\lambda(s) \qquad 0 \le t \le \theta \qquad (4.1.1)$$

$$\frac{d}{dt} I(t) + h(t)I(t) = -\lambda(s) \qquad \theta \le t \le T \qquad (4.1.2)$$

where $h(t)$ is as given in equation (2.1), with the initial conditions $I(0) = Q, I(T) = 0$.

Substituting $h(t)$ given in equation (2.1) in equations (4.1.1) and (4.1.2) and solving the differential equations, the on hand inventory at time t can be obtained as

$$I(t) = \lambda(s)\left\{(\theta - t) + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right) -(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right\}$$

$$0 \le t \le \theta \qquad (4.1.3)$$

$$I(t) = \lambda(s)\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$
$$\left\{\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right\}$$

$$\theta \le t \le T \qquad (4.1.4)$$

The stock loss due to deterioration in the interval (0, t) is

$$L(t) = I(0) - \lambda(s)t - I(t)$$
$$= \lambda(s)\left\{(\theta - t) + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right.$$
$$-\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$
$$\left.\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right\}$$

$$0 \le t \le T$$

The stock loss due to deterioration in the cycle of length T is

$$L(T) = \lambda(s)\left\{(\theta - T) + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}\right\} dt$$

The ordering quantity, Q in the cycle length T is

$$Q = L(T) + \lambda(s)T$$
$$= \lambda(s)\left[\theta + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1} dt\right] \qquad (4.1.5)$$

Let $K(T, s)$ be the total cost per unit time. Since the total cost is sum of the set up cost, cost of the units, the inventory holding cost, $K(T, s)$ becomes

$$K(T, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_{0}^{\theta} I(t)dt + \int_{\theta}^{T} I(t)dt\right] \qquad (4.1.6)$$

Substituting the values of $I(t)$ and Q given in equations (4.1.3), (4.1.4) and (4.1.5) in equation (4.1.6), we obtain $K(T, s)$ as

$$K(T, s) = \frac{A}{T} + \frac{C}{T}\lambda(s)\left[\theta + \int_{\theta}^{T} 1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)\right.$$
$$\left.-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} dt + \frac{h}{T}\lambda(s)\int_{0}^{\theta}\left\{\theta - t\right.$$
$$\left.+\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} du\right\} dt$$
$$+\frac{h}{T}\lambda(s)\int_{\theta}^{T}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$
$$\left.\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1} du\right\} dt \qquad (4.1.7)$$

Let $P(T, s)$ be the profit rate function. Since the profit rate function is the total revenue per unit time minus total cost per unit time, we have

$$P(T, s) = s\lambda(s) - K(T, s) \qquad (4.1.8)$$

International Journal of Scientific & Engineering Research Volume 5, Issue 7, July-2014
ISSN 2229-5518

Substituting the values of $K(T,s)$ given in equations (4.1.7) in (4.1.8), we get profit rate function as

$$P(T,s) = s\lambda(s) - \frac{A}{T} - \frac{C}{T}\lambda(s)\left\{\theta + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)\right.\right.$$

$$\left.\left.-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}dt\right\} - \frac{h}{T}\lambda(s)\int_{0}^{\theta}\left\{\theta - t\right.$$

$$\left.+\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$-\frac{h}{T}\lambda(s)\int_{\theta}^{T}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

(4.1.9)

4.2 OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section we obtain the optimal policies of the inventory system developed in section 4.1. To find the optimal values of T and s, we equate the first order partial derivatives of $P(T,s)$ given in equation (4.1.9) with respect to T and s to zero respectively. The conditions for obtaining optimality are

$$\frac{\partial}{\partial T}P(T,s) = 0; \qquad \frac{\partial}{\partial s}P(T,s) = 0;$$

$$\text{and } D = \begin{vmatrix} \frac{\partial^2}{\partial T^2}P(T,s) & \frac{\partial}{\partial T\partial s}P(T,s) \\ \frac{\partial}{\partial T\partial s}P(T,s) & \frac{\partial^2}{\partial s^2}P(T,s) \end{vmatrix} < 0$$

where D is the determent of Hessian matrix

Differentiating $P(T,s)$ given in equation (4.1.9) with respect to T and equating to zero, we get

$$\frac{A}{\lambda(s)} + C\theta$$

$$-C\left\{\int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right.$$

$$\left.-T\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}\right\}$$

$$-h\left[\int_{0}^{\theta}\left\{\theta - t + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)\right.\right.\right.$$

$$\left.\left.-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$-T\left[1-\alpha\left(1-\left(\frac{\theta}{T}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{T}\right)^{\beta_2}\right)\right]^{-1}\Bigg]$$

$$-h\int_{\theta}^{T}\left\{\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right.$$

$$\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$-T\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]$$

$$\left.\left[1-\alpha\left(1-\left(\frac{\theta}{T}\right)^{\beta_1}\right)(1-\alpha)\left(1-\left(\frac{\theta}{T}\right)^{\beta_2}\right)\right]^{-1}\right\}dt = 0$$

(4.2.1)

Differentiating $P(T,s)$ given in equation (4.1.9) with respect to s and equating to zero, we get

$$sT + \frac{\lambda(s)}{\lambda'(s)}T$$

$$-c\left\{\theta + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}dt\right\}$$

$$-h\int_{0}^{\theta}\left\{\theta - t + \int_{\theta}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)\right.\right.$$

$$\left.\left.-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt$$

$$-h\int_{\theta}^{T}\left\{\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]\right.$$

$$\left.\int_{t}^{T}\left[1-\alpha\left(1-\left(\frac{\theta}{u}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{u}\right)^{\beta_2}\right)\right]^{-1}du\right\}dt = 0$$

(4.2.2)

Solving equations (4.2.1) and (4.2.2) simultaneously, we obtain the optimal cycle length, $T^*$and optimal selling price, s^*.
The optimal ordering quantity, Q^* is obtained by substituting the optimal values of T and s in equation (4.1.5).

$$Q^* = \lambda(s^*)$$

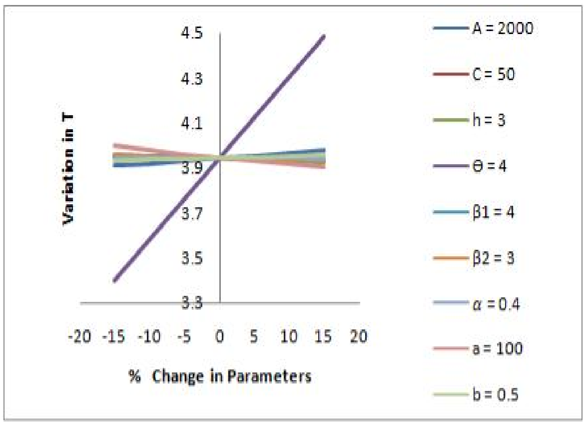
$$\left\{\theta + \int_{\theta}^{T^*}\left[1-\alpha\left(1-\left(\frac{\theta}{t}\right)^{\beta_1}\right)-(1-\alpha)\left(1-\left(\frac{\theta}{t}\right)^{\beta_2}\right)\right]^{-1}dt\right\}$$

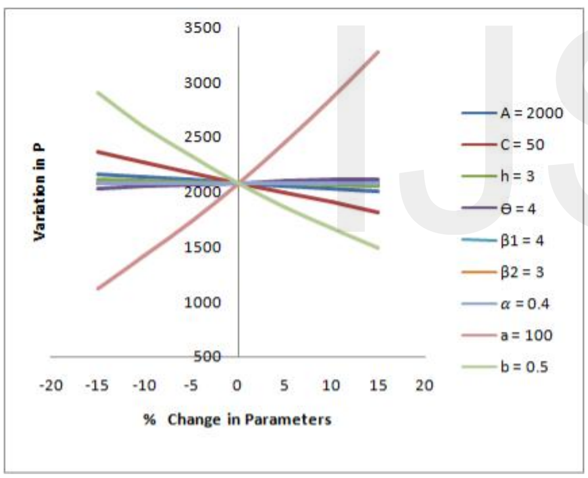
(4.2.3)

International Journal of Scientific & Engineering Research Volume 5, Issue 7, July-2014
ISSN 2229-5518

4.3 SENSITIVITY ANALYSIS OF THE MODEL

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2. The relationship between the parameters and optimal cycle length T^*, optimal profit P^*, and optimal ordering quantity Q^* is shown in Figure 3.





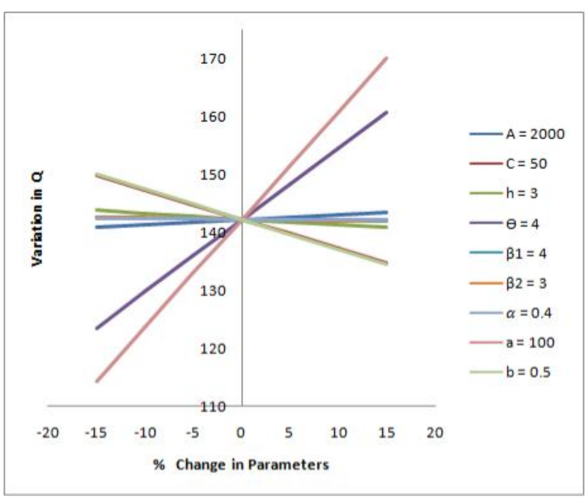


Fig 3: Relationship between optimal values and parameters without shortages

Table 2

Sensitivity analysis of the model-without shortages

Variation Parameter	Optimal values	%change in parameter						
		-15	-10	-5	0	5	10	15
A	T^*	3.9105	3.9219	3.9333	3.9445	3.9556	3.9667	3.9776
	s^*	127.9768	127.9949	128.0138	128.0334	128.0538	18.0748	128.0965
	K^*	2449.70711	2474.73458	2499.66645	2524.5044	2549.24987	2573.90456	2598.4699
	P^*	2158.9427	2133.4078	2107.9469	2082.5591	2057.2431	2031.9981	2006.8229
	Q^*	141.0238	141.4447	141.8642	142.2824	142.6993	143.1149	143.5293
C	T^*	3.9498	3.9478	3.9460	3.9445	3.9433	3.9422	3.9414
	s^*	124.2831	125.5329	126.7830	128.0334	129.2842	130.5352	131.7867
	K^*	2344.9991	2407.9781	2467.8132	2524.5044	2578.0511	2628.4530	2675.7095
	P^*	2360.1650	2266.0575	2173.5224	2082.5591	1993.1670	1905.3456	1819.0947
	Q^*	149.9250	147.3637	144.8167	142.2824	139.7593	137.2460	134.7416
H	T^*	3.9614	3.9556	3.9500	3.9445	3.9391	3.9338	3.9286
	s^*	127.6163	127.7554	127.8945	128.0334	128.1722	128.3109	128.4495
	K^*	2503.8457	2510.7879	2517.6739	2524.5044	2531.2798	2538.0006	2544.6675
	P^*	2114.8258	2104.0300	2093.2746	2082.5591	2071.8827	2061.2450	2050.6454
	Q^*	143.8044	143.2904	142.7832	142.2824	141.7880	141.2996	140.8171
Θ	T^*	3.4007	3.5825	3.7637	3.9445	4.1249	4.3048	4.4844
	s^*	127.6911	127.8016	127.9160	128.0334	128.1535	128.2758	128.3999
	K^*	2590.4281	2565.5081	2543.6849	2524.5044	2507.5961	2492.6553	2479.4282
	P^*	2026.1729	2048.0277	2066.6650	2082.5591	2096.0931	2107.5841	2117.2949
	Q^*	123.5280	129.8148	136.0661	142.2824	148.4643	154.6123	160.7268
β_1	T^*	3.9548	3.9512	3.9478	3.9445	3.9414	3.9384	3.9356
	s^*	128.0459	128.0415	128.0374	128.0334	128.0297	128.0261	128.0226
	K^*	2523.7342	2524.0039	2524.2598	2524.5044	2524.7379	2524.9613	2525.1752
	P^*	2082.9800	2082.83295	2082.69280	2082.5591	2082.43126	2082.3090	2082.1920
	Q^*	142.6502	142.52143	142.39901	142.2824	142.17123	142.0651	141.9635
β_2	T^*	3.9566	3.9524	3.9483	3.9445	3.9409	3.9375	3.9342
	s^*	128.0482	128.0430	128.0381	128.0334	128.0290	128.0248	128.0208
	K^*	2523.5991	2523.9176	2524.2189	2524.5044	2524.7752	2525.0327	2525.2778
	P^*	2083.0496	2082.8771	2082.7139	2082.5591	2082.4121	2082.2723	2082.1399
	Q^*	142.7170	142.5638	142.4192	142.2824	142.1529	142.0300	141.9132
α	T^*	3.9470	3.9461	3.9453	3.9445	3.9437	3.9429	3.9421
	s^*	128.0364	128.0354	128.0344	128.0334	128.0325	128.0315	128.0305
	K^*	2524.3193	2524.3816	2524.4433	2524.5044	2524.5648	2524.6246	2524.6839
	P^*	2082.6599	2082.6259	2082.5923	2082.5591	2082.5261	2082.4936	2082.4613
	Q^*	142.3708	142.3410	142.3116	142.2824	142.2536	142.2250	142.1967
a	T^*	4.0022	3.9802	3.9611	3.9445	3.9299	3.9169	3.9053
	s^*	113.1484	118.1016	123.0641	128.0334	133.0081	137.9869	142.9689
	K^*	2100.0022	2241.9379	2383.4035	2524.5044	2665.3165	2805.8962	2946.2864
	P^*	1116.3319	1413.2113	1735.3003	2082.5591	2454.9572	2852.4714	3275.0831
	Q^*	114.3087	123.6488	132.9720	142.28241	151.5831	160.8717	170.1632
b	T^*	3.9321	3.9361	3.9402	3.9445	3.9489	3.9536	3.9583
	s^*	145.6589	139.1298	133.2890	128.0334	123.2795	118.9590	115.0153
	K^*	2642.8533	2603.4432	2563.9945	2524.5044	2484.9696	2445.3870	2405.7527
	P^*	2906.0181	2598.8420	2326.0755	2082.5591	1864.1162	1667.3299	1489.3775
	Q^*	150.0989	147.4953	144.8899	142.2824	139.6727	137.0606	134.4460

It is observed that the costs are having significant influence on the cycle length, profit per unit time, Quantity. As the ordering cost A increases, optimal cycle length T^*, optimal selling price s^*, and optimal ordering quantity Q^* are increasing and the optimal profit P^*, is decreasing. When the cost per unit C increases, optimal selling price s^*, and optimal profit P^* are increasing and , optimal cycle length T^*, and optimal ordering quantity $Q^*$are decreasing. When the holding cost h increases, optimal cycle length T^*, optimal profit P^*, and optimal ordering quantity Q^* are decreasing and optimal selling price s^*, is increasing.

As the scale parameter θ increases, optimal cycle length T^*, optimal selling price s^*, optimal profit P^*, and optimal ordering quantity Q^*, are increasing. As the shape parameter β_1 increases optimal cycle length T^*, optimal selling price s^*, optimal profit P^*, and optimal ordering quantity Q^*, are decreasing. As the shape parameter β_2 increases optimal cycle length T^*, optimal selling price s^*, optimal profit P^*, and optimal ordering quantity Q^*, are decreasing. As the mixing parameter 'α' increases optimal cycle length T^*, optimal selling price s^*, optimal profit P^*, and optimal ordering quantity Q^*, are decreasing.

As the demand parameter 'a' increases optimal cycle length T^*, optimal profit P^*, and optimal ordering quantity Q^*, are decreasing and optimal selling price s^*, is increasing. As the demand parameter ' b' increases optimal selling price

International Journal of Scientific & Engineering Research Volume 5, Issue 7, July-2014
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s^*, optimal profit P^*, and optimal ordering quantity Q^*, are decreasing and optimal cycle length T^*, is increasing.

5 Conclusion

This paper deals with the inventory model for deteriorating items with two component mixture of Pareto distribution. Here it is assumed that the life time of the commodity is random and follows a two component mixture of Pareto distribution. The two component mixture of Pareto distribution characterizes the heterogeneous nature of life time of the commodity. In many practical situations the procurement is done from different sources / vendors. For the first time, the utility of two component mixture of Pareto distribution in inventory models is done because of its close reality to the practical situations. Here it is considered that the demand is a function of selling price. The optimal pricing and ordering policies of the model are derived. The sensitivity analysis of the model reveals that the deteriorating distribution parameter and costs have significant influence on the pricing and ordering policies of the model. It is also observed that this model includes the inventory model for deteriorating items given by Srinivasa Rao et.al [12] as a particular case for specific value of the mixing parameter. By suitable estimation of the cost and deteriorating distribution of the parameters, the operational managers of market yards / ware houses / production processes can obtain the optimal schedules using historical data. It is also possible to develop inventory model for deteriorating items with heterogeneous life times and stock dependent demand which can be taken up elsewhere.

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